Contents

1 Introduction

This document is meant to provide a short introduction to the structure of the code of for the discrete directional Gabor transform, ddg2d, and its inverse, iddg2d. For more details on discrete directional Gabor systems, see [1].

2 Overview of Discrete Directional Gabor Systems

From [1], we have conditions on a discrete set $\Lambda \subset \mathbb{R} \times \mathbb{R} \times \mathcal{S}^{d-1}$ such that the collection

$$\{g(u \cdot x - t)e^{i2\pi m u \cdot x}\}_{(m,t,u)\in\Lambda}$$

forms a frame for $L^2(\mathbb{R}^d)$ whose support is contained in $[-1/2, 1/2]^d$. In particular for any $\omega \in \mathbb{R}^+$ we have that

$$\Lambda_\omega = \{(m, n, u) : (u, m) \in \Gamma, n \in \omega\mathbb{Z}\}$$

where $\Gamma \subset \mathcal{S}^{d-1} \times \mathbb{R}$ is such that $\psi : \Gamma \to \mathbb{Z}^d$ given by $(u, m) \mapsto mu$ is a bijection, is one such discrete set. One simple construction of Γ is as follows:

$$\Gamma = \left\{\left(\frac{a}{||a||_2}, ||a||_2\right)\right\}_{a\in\mathbb{Z}^d\setminus\{0\}} \cup \{(0, 1)\}.$$

3 Overview of Code

Note that currently there is only code for $d = 2$ so in all of our discussion which follows, we will assume that $d = 2$.

3.1 `gen_lambda`

This function generates a MATLAB structure which represents a finite subset of the set $\Lambda_\omega = \{(m, n, u) : (u, m) \in \Gamma, n \in \omega\mathbb{Z}\}$ described above. While it is immediately obvious how to choose n, constructing a finite subset of Γ requires a little more thought. Instead explicitly constructing Γ, it is more straightforward to pick some $A \subset \mathbb{Z}^2$ finite then generate a Γ_A based on A. More concretely, for A fixed we define

$$\Gamma_A = \left\{\left(\frac{a}{||a||_2}, ||a||_2\right)\right\}_{a\in A\setminus(0,0)} \cup B$$

where

$$B = \begin{cases} \emptyset & (0, 0) \notin A \\ (0, 1) & (0, 0) \in A \end{cases}$$

Currently, there are two ways that subsets A can be specified: 'square' and 'specify'. They differ as follows

- **square**: Takes input in the form of a cell with three elements. The first element is a range of n values, the second element is a range of x values, and the third element is a range of y values. The subset A is specified by the Cartesian product of the range of x values with the range of y values.
- **specify**: Takes input in the form of a cell with two elements. The first element is a range of n values, the second element is a list of all the points in A.

In both cases, our finite subset of Λ_ω is the Cartesian product of the range of n values with Γ_A.

3.2 `analysis_synthesis_op`, `ddg2d`, and `iddg2d`

The discrete directional Gabor system is fully discretized, we can represent the linear transform which transforms an image to its discrete directional Gabor representation (the so-called 'analysis' operator) as a matrix, call it S. The dual of this transform is simply the conjugate transpose of S (the so called 'synthesis' operator). Because of the similarity between S and its S^* over 90% of the code for implementing these two operators are the same, so we have combined them into one function `analysis_synthesis_op`.

Suppose we have an input image f, the `ddg2d` function calculates Sf. To recover f from Sf in our implementation of `iddg2d`, we calculate S^*Sf then apply conjugate gradient (MATLAB's `pcg` routine) to find the inverse of S^*S.

References

[1] Wojciech Czaja, Benjamin Manning, James M. Murphy, and Kevin Stubbs. Discrete directional Gabor frames. *Applied and Computational Harmonic Analysis*, to appear.