Homework 1<br>MATH 165 - Fall 2020<br>Tufts University, Department of Mathematics

Due: September 17, 2020

## 1. Book Questions

Grinstead and Snell: Section 1.2, \#14, \#18, \#20, \#23, \#28

## 2. Supplemental Question (Coupon Collector's Problem, Part 1)

Let $m \in \mathbb{Z}_{+}$, and let $X_{m}$ a random variable with uniform distribution on $\{1,2,3, \ldots, m\}$. Define $Y_{m, n}$ to be the random variable equal to the number of unique elements of $\{1,2,3, \ldots, m\}$ generated from $X_{m}$ after $n$ i.i.d. samples. Note that in general, $n \neq m$.
(a) Compute $\mathbb{P}\left(Y_{m, m}=m\right)$.
(b) Fix $m \in \mathbb{Z}_{+}$. Compute $\lim _{n \rightarrow \infty} \mathbb{P}\left(Y_{m, n}=m\right)$.

