Homework 1 MATH 165 - Fall 2020 Tufts University, Department of Mathematics Due: September 17, 2020

1. BOOK QUESTIONS

Grinstead and Snell: Section 1.2, #14, #18, #20, #23, #28

2. SUPPLEMENTAL QUESTION (COUPON COLLECTOR'S PROBLEM, PART 1)

Let $m \in \mathbb{Z}_+$, and let X_m a random variable with uniform distribution on $\{1, 2, 3, \ldots, m\}$. Define $Y_{m,n}$ to be the random variable equal to the number of *unique* elements of $\{1, 2, 3, \ldots, m\}$ generated from X_m after n i.i.d. samples. Note that in general, $n \neq m$.

(a) Compute $\mathbb{P}(Y_{m,m} = m)$.

(b) Fix $m \in \mathbb{Z}_+$. Compute $\lim_{n \to \infty} \mathbb{P}(Y_{m,n} = m)$.