Homework 10
MATH 165 - Fall 2020
Tufts University, Department of Mathematics
Due: November 19, 2020

1. Book Questions

Grinstead and Snell: Section 9.1, #1, #8, #15; 9.2, #2, #15

2. Supplemental Question (Convergence Rate of CLT)

Recall the following formulation of the central limit theorem.

Theorem. *(CLT) Let* $\{X_i\}_{i=1}^n$ *be i.i.d. random variables with common mean* μ *and common variance* $\sigma^2 > 0$. *Let* $S_n^* = \frac{1}{\sqrt{n\sigma^2}}\left(\sum_{i=1}^n X_i - n\mu\right)$. *Then for all* $a > b$, $\lim_{n\to\infty} \mathbb{P}(a < S_n^* < b) = \frac{1}{\sqrt{2\pi}}\int_a^b \exp(-z^2/2)dz$.

We are interested in how fast the limit on the left converges to the integral on the right.

(a) Let X_i be uniform on $[0, 1]$. Compute μ, σ^2.

(b) Using the results in (a), for the $n \in \{100, 200, 300, 400, \dots, 10000\}$, compute 100 random samples from S_n^*, and use Monte Carlo to estimate $\mathbb{P}(-1 < S_n^* < 1)$ as a function of n.

(c) Using the results in (b), plot the estimated error

$$E_n = \frac{\left|\mathbb{P}(-1 < S_n^* < 1) - \frac{1}{\sqrt{2\pi}}\int_{-1}^1 \exp(-z^2/2)dz\right|}{\frac{1}{\sqrt{2\pi}}\int_{-1}^1 \exp(-z^2/2)dz}$$

as a function of n.

(d) Can you describe the shape of the error curve? *Hint: Consider the log-log plot of* $\log(E_n)$ *versus* $\log(n)$.