Homework 11
MATH 165 - Fall 2020
Tufts University, Department of Mathematics
Due: December 3, 2020

1. BOOK QUESTIONS

Grinstead and Snell: Section 9.3, #1, #7; 10.1, #7, #11; 10.3, #10

2. SUPPLEMENTAL QUESTION (INFINITE PRODUCTS OF UNIFORM RANDOM VARIABLES)

Let $\{X_i\}_{i=1}^n$ be i.i.d. uniform random variables on $[0,1]$.

(a) Show that the Central Limit Theorem applies to the averages $\dfrac{1}{n}\sum_{i=1}^n \log(X_i)$. What can be said (after a suitable normalization) as $n \to \infty$?

(b) Use the result in (a) to explain the asymptotic behavior as $n \to \infty$ of the random variables
$$Y_n := \left(\prod_{i=1}^n X_i\right)^{\frac{1}{n}}.$$