Homework 12
MATH 165 - Fall 2020
Tufts University, Department of Mathematics
Due: December 10, 2020

1. Book Questions

Grinstead and Snell: Section 11.1, #7, #14; 11.2, #9, #27; 11.3, #2

2. Supplemental Question (Markov Chains on Bipartite Graphs)

Let $S = \{s_i\}_{i=1}^{2n}$ be $2n$ states and consider a matrix $P \in \mathbb{R}^{2n \times 2n}$ with

$$P_{ij} = \begin{cases} 0 & \text{if } i \in \{1, 2, \ldots, n\} \text{ and } j \in \{1, 2, \ldots, n\} \\ 1/n & \text{if } i \in \{1, 2, \ldots, n\} \text{ and } j \in \{n+1, n+2, \ldots, 2n\} \\ 1/n & \text{if } i \in \{n+1, n+2, \ldots, 2n\} \text{ and } j \in \{1, 2, \ldots, n\} \\ 0 & \text{if } i \in \{n+1, n+2, \ldots, 2n\} \text{ and } j \in \{n+1, n+2, \ldots, 2n\} \end{cases}$$

(a) Show that P is in fact a Markov chain on S where $\mathbb{P}(s_i \to s_j) = P_{ij}$.

(b) Explain the dynamics encoded by P intuitively. In what sense might this Markov chain be reasonably called periodic?

(c) Show that $\lim_{t \to \infty} P^t$ does not exist.