

**Homework 3**  
MATH 165 - Fall 2020  
Tufts University, Department of Mathematics  
Due: October 1, 2020

1. BOOK QUESTIONS

Grinstead and Snell: Section 3.1 #8, #9 #12; Section 3.2, #6, #35

2. SUPPLEMENTAL QUESTION (RANDOM SUBSETS)

Let  $A = \{1, 2, \dots, n\}$ . Define a *random subset*  $B$  of  $A$  by setting  $\mathbb{P}(k \in B) = \frac{1}{2}$  for all  $k \in A$  (this is the uniform distribution over subsets of  $A$ ). Let  $X$  be the random variable equal to the cardinality of  $B$ , and let  $m_X(x) = \mathbb{P}(|B| = x)$  be its distribution function.

- (a) For what value(s) of  $x$  is  $m_X$  maximal when  $n$  is odd?
- (b) For what value(s) of  $x$  is  $m_X$  maximal when  $n$  is even?
- (c) Use Stirling's approximation to prove that  $\lim_{n \rightarrow \infty} \left( \max_{x \in \{1, 2, \dots, n\}} m_X(x) \right) = 0$ .