Homework 5
MATH 165 - Fall 2020
Tufts University, Department of Mathematics
Due: October 15, 2020

1. Book Questions

Grinstead and Snell: Section 4.3 #1; Section 5.1, #6, #24; Section 5.2, #9, #37

2. Supplemental Question (Benford Distributions)

Let N be a fixed positive integer. A random variable X on $\{1, 2, \dots, N-1\}$ has a *Benford distribution* if its distribution function is $m_X(k) = \mathbb{P}(X = k) = \log_N(k+1) - \log_N(k)$.

(a) Show that m_X actually defines a probability distribution on $\{1, 2, \dots, N-1\}$.

(b) Let $\{x_i\}_{i=1}^{1000}$ be an i.i.d. sample from Unif([0, 1]). Let $y_i = 10^{x_i}$. Show empirically that the *leading digit (i.e. first non-zero digit)* of the $\{y_i\}_{i=1}^{1000}$ is approximately a Benford distribution with $N = 10$.

(c) What about the second digit of the $\{y_i\}_{i=1}^{1000}$?

(d) Give an intuitive explanation for (b).

(e) What happens as N increases?