Homework 7 MATH 165 - Fall 2020 Tufts University, Department of Mathematics Due: October 29, 2020

1. BOOK QUESTIONS

Grinstead and Snell: Section 6.2 #11, #23; 6.3 #13, #27

2. SUPPLEMENTAL QUESTION (CAUCHY RANDOM VARIABLES AND HIGHER ORDER MOMENTS)

Let X be a continuous \mathbb{R} -valued random variable with density f(x). Define, for $p \in \{0, 1, 2, 3, ...\}$, the moment of order p of X to be

$$\mathbb{E}(X^p) = \int_{-\infty}^{\infty} x^p f(x) dx.$$

Notice that the moment of order 1 is just the expected value. Moreover, the moment of order 2 is the variance if X has expected value 0.

- (a) Let X be a Cauchy random variable, so that X has density $f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}$. Show that for all $p \in \{1, 2, 3, ...\}$, the moment of order p of X does not exist.
- (b) For each $q \in \{0, 1, 2, 3...\}$, construct a random variable X (depending on q) such that the following two properties hold simultaneously:
 - (a) All moments of order $p \leq q$ exist and are finite.
 - (b) All moments of order p > q do not exist.