

Homework 5

MATH 166 - Spring 2023

Tufts University, Department of Mathematics

Instructor: James M. Murphy

Due: February 28, 2023

1. BOOK QUESTIONS

Wasserman: Chapter 9: #3 (in part (c), ignore the part about the parametric bootstrap. Hint: look at example 9.29), #5 (ignore the part about Fischer information)

SUPPLEMENTAL QUESTION 1 (ASYMPTOTIC UNBIASEDNESS)

We say an estimator $\hat{\theta}_n$ is *asymptotically unbiased* for θ if $\lim_{n \rightarrow \infty} \mathbb{E}(\hat{\theta}_n) = \theta$, where as usual the expectation is taken over the random sample. Let x_1, \dots, x_n be an i.i.d. sample from $\text{Unif}(0, \theta)$. Recall that the MLE estimator for θ is $\hat{\theta}_n = \max_{1 \leq i \leq n} x_i$.

- (a) Show $\hat{\theta}_n$ is biased for every n .
- (b) Show $\hat{\theta}_n$ is asymptotically unbiased.

SUPPLEMENTAL QUESTION 2 (PROPERTIES OF KL DIVERGENCES)

The Kullback-Leibler distance is not a metric in the traditional sense. We will investigate some of its properties below. Let f, g be any probability density functions.

- (a) Show $D_{KL}(f, f) = 0$.
- (b) Show $D_{KL}(f, g) \geq 0$ (Hint: $\log\left(\frac{1}{y}\right) \geq 1 - y$ for all y).