MATH 166 Statistics - Spring 2023
Tufts University, Department of Mathematics
Instructor: James M. Murphy
Due: March 14, 2023
HW7: Practice Exam 1

Instructions: This exam has 3 questions and is out of a total of 30 points. Each question is worth 10 points. No graphing calculators, books, or notes are allowed. Be sure to show all work for all problems. No credit will be given for answers without work shown. If you do not have enough room in the space provided you may use additional paper. Be sure to clearly label each problem and attach them to the exam. You have 75 minutes. Good luck! :-)

Your Printed Name: ______________________________________

Problem	Score
1	
2	
3	
Total	

Academic Honesty Certification:

I certify that I have taken this exam without the aid of unauthorized people or objects.

Signature: ____ Date: ____________________________

Question 1

Let X be a random variable with finite expectation μ. Let $x_1, \ldots, x_n$ be an i.i.d. sample from X.

(a) Show the estimator $\hat{\mu}(x_1, \ldots, x_n) = \frac{1}{n-1} \sum_{i=1}^{n} x_i$ is biased for μ.

(b) Show the estimator $\hat{\mu}(x_1, \ldots, x_n) = x_1$ is unbiased for μ.

(c) Suppose n is large. Which estimator do you prefer and why?

Question 2

Let X be an exponential random variable with unknown parameter $\lambda > 0$; recall this means X has density

$$f(x) = \begin{cases} \lambda \exp(-\lambda x), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

Suppose we observe an i.i.d. sample $x_1, \ldots, x_n$ from X.

(a) Estimate λ using maximum likelihood.

(b) Estimate λ using the method of moments. *Hint: you may use, without proof, the fact that the expected value of X is* $1/\lambda$.

Question 3

Let $x_1, \dots x_n$ be an i.i.d. sample from a uniform distribution on $(0, \theta)$ for some unknown parameter θ.

(a) Show that the maximum likelihood estimator for θ is $\hat{\theta}(x_1, \dots, x_n) = \max_{i=1,\dots n} x_i$.

(b) Show that this estimator is consistent. *Hint: show and then use the fact that for any* $\epsilon > 0$, $\mathbb{P}(|x_1 - \theta| > \epsilon) = \mathbb{P}(x_1 \in [0, \theta - \epsilon)) = \frac{\theta - \epsilon}{\theta}$.