Homework 8
MATH 166 - Spring 2023
Tufts University, Department of Mathematics
Instructor: James M. Murphy
Due: March 30, 2023

1. Book Questions

Wasserman: Chapter 10: #6, #10

Supplemental Question 1 (Properties of χ^2 distributions)

Let $Y_k = \sum_{i=1}^{k} Z_i^2$ be the sum of k independent squared standard Gaussians (i.e., $Z \sim \mathcal{N}(0,1)$). We call such a random variable a χ^2 *distribution with k degrees of freedom.*

(a) Compute $\mathbb{E}(Y_k)$.

(b) Compute $\text{Var}(Y_k)$.

Supplemental Question 2 (Testing for Uniformity)

Let X be uniform on $[0,1]$. Fix K a positive integer, and sample $\{x_i\}_{i=1}^{n}$ i.i.d. fron X. For $k = 1, 2, \ldots, K$, define

$$Y_k = \left| \left\{ x_i \mid x_i \in \left[\frac{k-1}{K}, \frac{k}{K} \right] \right\} \right|$$

to be the random variable that counts the number of observations landing in the interval $[\frac{k-1}{K}, \frac{k}{K}]$.

(a) Show that the random vector $(Y_1, Y_2, \ldots, Y_K)$ defines a multinomial distribution. What are it's parameters?

(b) Develop a hypothesis test framework for testing whether data in $[0,1]$ comes from a uniform distribution, based on the observations in (a).

(c) The role of K is crucial to making (b) work. Discuss how to select a good choice of K, and what can happen if K is taken too small or too large.